PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Some Observations on the Amount of Light Reflected and Transmitted by Certain Kinds of Glass

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VIII. *Some Observations on the Amount of Light Reflected and Transmitted by Certain Kinds of Glass.*

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[Plate 8.]

Introduction.

Although for both theoretical and practical purposes it is important to know the amount of light reflected from the surface of glass, and the loss which light suffers in passing through glass, but few accurate experiments appear to have been made on this subject.

Dr. Robinson, in the report on the Melbourne telescope ('Phil. Trans.,' 1869, p. 127), gives an account of experiments made by Lord Rosse, Mr. Grubb, and himself to determine the amount of light transmitted by telescopic object glasses, and through various kinds of flint and crown glass; he states that Lord Rosse's and Mr. Grubb's experiments were made with a Bunsen's photometer, and his own with a Zöllner's photometer. Dr. Robinson assumed the truth of Fresnel's formulæ, and then calculated the values of the extinction coefficients ϵ^{-nt} from the expression $n = (\log \rho^2 - \log I)/(t \times \text{modulus})$, where I is the intensity of the emergent light, t the thickness of the glass, and ρ^2 the coefficient giving the amount which escapes reflection at the two surfaces.

Mr. Rood published in the 'American Journal of Science' for 1870 (vol. 50, p. 1) an account of some observations he had made, with a modification of Bunsen's photometer, of the loss which light suffers in passing through crown glass. He states that the formulæ for reflection were originally given by Young, and obtained subsequently by Poisson and Fresnel, but believes that no accurate experiments have ever been made to test their truth. He used two plates of glass, ·15 mm. and 1·677 mm. thick, and assumed that with such thin plates the loss of light was practically entirely due to reflection. He found that with one plate, of which the index was 1·5236, and which therefore should have transmitted 91·736 per cent. of

30.5.89

the incident light, 91·440 per cent. actually passed through; and with the other, of which the index was 1·5225, 91·155 was transmitted, instead of 91·763.

VOGEL published in the 'Berliner Monatsberichte' for 1877 (p. 138) some determinations of the absorption of crown, and of two kinds of flint, glass for different parts of the spectrum, but states that, as the three glasses were not equally well polished, no conclusion can be drawn as to their relative merits.

M. ALLARD ("L'Intensité des Phares," 'Annales des Ponts et Chaussées,' 1876, p. 31) states that the absorption of light by the glass which it passes through is given "par une formule exponentielle, mais on peut sans grande erreur la supposer proportionelle à l'épaisseur et l'évaluer à raison de 0·03 par centimètre de verre traversé," but does not give an account of any experiments made to determine the amount absorbed.

Since the experiments described in this paper were commenced Lord RAYLEIGH has published ('Roy. Soc. Proc.,' vol. 41, p. 275) an account of some determinations he has made of the intensity of the light reflected from glass at a nearly perpendicular incidence; he came to the conclusion that "recently polished glass surfaces have a reflecting power differing not more than 1 or 2 per cent. from that given by FRESNEL'S formula; but that after some months or years the reflection may fall off from 10 to 30 per cent., and that without any apparent tarnish."

The experiments of which I have the honour of presenting an account to the Royal Society were commenced in order to determine, if possible, the amount of light lost by transmission through glass, without assuming the truth of the formulæ for reflection, and also to determine experimentally the amount reflected from the surface of the glass.

It was thought that one way in which this might be effected would be by taking plates of the same kind of glass, but of different thicknesses, and observing the amount of light which passed through; the reflection from the first surface would be the same in all cases, whilst that from the second would only differ slightly in amount, as, owing to the increased absorption of the thicker pieces, less light would reach the second surface; but, the absorption being small, and photometric methods not very exact, it was thought that this would hardly produce any sensible difference in the results.

It was also hoped that it might be found possible to determine the reflected light directly, since, as Lord RAYLEIGH has pointed out (*loc. cit.*) with reference to Professor ROOD'S experiments, any error in the measured amount of the light transmitted would give rise to a very much greater error in the estimated amount of the reflected light. This was subsequently accomplished by measuring the relative intensities of the illumination produced by two Argand gas flames, when the light from both fell directly on the photometric surfaces, and when the light from one fell directly, whilst that from the other reached the photometer after reflection from the surface of a piece of glass.

Experiments were also made to ascertain whether repolishing altered in any way the reflective power of the glass; and the polarising angles before, and after, repolishing were also determined.

The observations were made with five plates of Messrs. CHANCE's "lighthouse" glass, for which the author is indebted to Mr. KENWARD, the plates being 6·5 mm., 11·5 mm., 15 mm., 18·5 mm., and 24·3 mm. thick respectively, and with different thicknesses of Messrs. FIELD's "ordinary dense flint." A block of this glass was procured from Mr. HILGER, by whom a slice 7 mm. thick was cut off and polished, and then the remainder of the piece formed into a rectangular block measuring 91·3 mm. × 69·5 mm. × 49 mm., and the six faces carefully polished.

Some measurements were also made with a piece of ordinary plate glass 6 mm. thick.

The ordinary plate glass was green when seen edgewise, Messrs. CHANCE's glass was slightly green when viewed in the same manner, and the flint glass was distinctly yellow by daylight.

Mr. KENWARD states that "the lighthouse glass is of a special mixture, varying slightly from time to time; it is of the nature of hard crown glass." The average refractive index for the sodium line of Messrs. CHANCE's hard crown is stated by them to be 1·5172, and of their soft crown 1·5146; and in a letter which accompanied the glass its index was said to be "about 1·51 or 1·52."

The refractive indices for the sodium line of the crown glass, the flint glass, and the plate glass used in these experiments were determined in the ordinary way with small prisms made of each kind of glass by Mr. HILGER; the values found were:—

Crown glass	1·5145,
Flint glass	1·6330,
Plate glass	1·5274.

The indices of the crown and plate glass were also determined with the wedges of these two kinds of glass used for the reflection experiments; that for the crown was found to be 1·5137, and of the flint 1·6385; the refracting angles of these two wedges being only 9° 39′ 45″ and 9° 51′ 19″, whilst those of the small prisms were 59° 45′ 19″ and 59° 45′ 17″, it seemed probable that the values obtained with the latter were the most accurate, and they were therefore taken as the true values of the indices.

The account of the experiments is given in Part I., and the results deduced from them in Part II., the account of the experiments being divided into six sections:— 1. Amount of light transmitted. 2. Amount of light reflected at a nearly perpendicular incidence. 3. Amount of light reflected at a nearly perpendicular incidence after repolishing. 4. Amount of light reflected at various incidences between 0° and 90° by the crown glass before and after repolishing. 5. Amount of light transmitted after repolishing. 6. Values of the polarising angles before and after repolishing.

PART I.

Section I.—Determination of the Amount of Light Transmitted.

A photometric arrangement similar to the one described in PICKERING'S 'Physical Manipulations,' vol. 1, p. 132, was used for the greater number of these determinations (see Plate 8, fig. 1). Two similar pieces of looking glass 125 mm. × 125 mm. were fixed vertically at the ends of a horizontal board rather more than 2 metres long and 27 cm. wide, the mirrors being 2 metres apart; a small Argand gas burner, giving a flame 15 mm. in diameter, was fixed opposite the middle point of the line joining the centres of the mirrors, and at a horizontal distance of 20 cm. from the line; the mirrors were so adjusted that they reflected the light of the lamp towards each other.

A block of wood, resting on four small metal rollers and guided by two strips of wood fixed 7·8 cm. apart on the upper surface of the horizontal board, carried the photometer; an index was fixed to the wood block, and a scale, divided into millimetres, to the board, the zero being at one of the mirrors.

The photometer of which a description was given in 'Roy. Soc. Proc.,' vol. 35, p. 27, and 'Phil. Mag.,' series 5, vol. 15, p. 423, was used (Plate 8, fig. 1A); it consisted essentially of two pieces of white paper so placed that, whilst each was illuminated by one only of the two lights to be compared, both were visible to the observer. Three pieces of wood were screwed to the block, and between these the photometer was placed; this arrangement, whilst permitting the photometer to be reversed, so that the light from each of the two mirrors could be made to fall first on the one and then on the other paper, ensured its always being replaced in the same position.

The whole arrangement was optically equivalent to two exactly similar sources of light which always retained the same relative intensity, and were at a distance apart of a little more than twice the distance between the two mirrors.

A wooden screen was placed between the lamp and the observer, and, in order to cut off stray light reflected from the horizontal board, screens, with apertures in them rather larger than the apertures in the photometer box, were placed on either side of it, and between it and the mirrors; the edges of the apertures in the screens being formed of metal filed to a feather edge, and then, together with all the wood work, painted a dead black.

Assuming that the two sides of the Argand flame were equally bright, that the reflective powers of the two looking glasses were equal, and that there was no stray light, or at least that there was an equal amount from either side, then at a point midway between the two mirrors the intensities of the light reflected from the mirrors would necessarily have been equal, and, in order to determine the amount of light which passed through any transparent substance, it would merely have been necessary to interpose the substance in question between one of the mirrors and the photometer, and determine, by means of the photometer, the new position of equality.

But, as such assumptions might be erroneous, the experiments were actually made in the following manner :—First, the glass plate was fixed on one side of the photometer, whose position was altered until both papers appeared equally bright; six readings having been thus made, the glass was then placed on the other side, and six more readings made. The glass was then replaced in its original position, and six more readings made, and so on. After thirty-six readings the photometer was reversed, and another set of thirty-six readings made in the same manner. In making the readings, the photometer was first placed too much to the right, and then moved to the left till the point at which the illumination was equal was reached; it was then pushed to the left, and gradually moved back to the right, till the papers appeared equally bright, being thus brought to the position of equality alternately from one side and from the other.

In order to prevent the possibility of any light reflected from the edges of the plates reaching the photometer, the plates were placed either close to it or close to one of the screens. For two sets of readings the glass was placed against the opening in the box enclosing the photometer, and for one set against the opening in the screen, except in the case of the thick piece of flint glass, which was too large and heavy to be carried by the block on which the photometer rested, and which was therefore always placed against one of the screens. The position of the glass made no difference in the measurements.

The percentage transmitted of light falling upon the glass was calculated in the following manner:—Half the difference between the means of each set of six observations with the plate on either side of the photometer was taken; half the distance between the two mirrors, plus or minus this quantity, together with the distance between the lamp and the mirrors, gave the two distances from the lamp at which there was equality of illumination. The mirrors being 200 cm. apart and the lamp at a horizontal distance of 20 cm. from the middle of the line which joined their centres, the half difference was added to or subtracted from 202.*

Calling these distances a and b, a being the lesser, a correction x had to be made in the value of a for the optical shortening of the path of the light due to its passage through the glass; this was calculated by the ordinary formula $x = e(1 - 1/n)$, where e is the thickness of the plate, and n its refractive index, the distance of the lamp from the glass being sufficiently great to allow the formula for perpendicular incidence to be used without introducing any sensible error.

The percentage amount of light transmitted was given by the expression $100\,(a - x)^2/b^2$.

* Strictly speaking, the geometrical, and not the arithmetical, mean of the readings should have been taken, as the intensity of light varies inversely as the square of the distance. Calling the two sources of light, to which the lamp and mirrors were equivalent, m and n, and the distance between them x, the

The surfaces of the glass plates were always cleaned with a wash-leather immediately before the plates were used, but, in order to ascertain whether this was sufficient, a number of readings were made with a piece of plate glass treated in this way, and then the plate was cleaned with strong nitric acid, washed with water, with alcohol, and again with water, dried with a clean cloth, and finally rubbed with a wash-leather. The means of the twelve readings made immediately before, and immediately after, the plate had been so treated were identical.

Table I. gives the readings made with the 24·3 mm. plate of crown glass. The observations made with the other pieces of glass were about as concordant.

two positions of the photometer in which there is equality x_1 and x_2, and the coefficient of transparency for the particular piece of glass k, then

$$\frac{mk}{x_1^2} = \frac{n}{(x - x_1)^2}, \qquad \text{and} \qquad \frac{nk}{(x - x_2)^2} = \frac{m}{x_2^2},$$

then

$$k = \frac{nx_1^2}{m(x - x_1)^2}, \qquad \text{and } k = \frac{m(x - x_2)^2}{nx_2^2},$$

$$k^2 = \frac{x_1^2 (x - x_2)^2}{x_2^2 (x - x_1)^2},$$

$$k = \frac{x_1 (x - x_2)}{x_2 (x - x_1)}.$$

The difference, however, between the values of k obtained by taking the quotient of the squares of the geometrical means of the readings, and by taking the quotient of the squares of the arithmetical means,

$$\left(\frac{\frac{x_1 + (x - x_2)}{2}}{\frac{x_2 + (x - x_1)}{2}}\right)^2,$$

was very small, and, as the readings themselves could not be made with any very great degree of accuracy, the simpler process was used for calculating out the results.

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TABLE I.

A.					
cm.	cm.	cm.	cm.	cm.	cm.
97·1	110·8	96·1	110·4	96·1	109·6
97·3	111·1	96·8	110·2	96·8	110·0
96·0	110·4	96·7	109·6	96·5	109·3
96·3	111·2	96·4	109·8	96·5	110·6
96·3	109·8	95·9	109·3	95·8	109·5
96·7	110·7	96·5	110·0	95·8	109·7
Mean 96·6	110·7	96·4	109·9	96·2	109·8
B.					
94·3	106·3	93·5	107·2	94·6	107·1
94·2	107·3	94 2	106·4	94·5	106·7
93·6	106·5	93·7	106·5	94·0	107·1
94·2	106·8	93·6	106·9	94·3	107·5
94·0	106·8	94·1	106·8	94·4	106·8
94·4	106·9	94·8	106·1	94·4	107·1
Mean 94·1	106·7	94·0	106·3	94·4	107·0

Table II. gives the mean readings (1st column), their half differences (2nd), the values of $a - x$ and b (3rd and 4th), and the percentage amount of light transmitted (5th), A and B being the readings made in the two positions of the photometer.

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TABLE II.

Crown Glass. 6·5 mm. plate. $x = 2$ mm.

Readings.	Half differences.	$a - x$.	b.	Percentage of light transmitted.	Percentage of light transmitted. Mean of A and B
A.					
cm.	cm.				
97·7–106·1	4·2	197·6	206·2	91·84	
97·1–106·3	4·6	197·2	206·6	91·11	
96·7–105·4	4·3	197·5	206·3	91·65	
				91·53	
B.					
94·7–103·4	4·3	197·5	206·3	91·65	
95·1–104·0	4·4	197·4	206·4	91·47	
95·1–104·1	4·5	197·3	206·5	91·29	
				91·47	91·50

Crown Glass. 11·5 mm. plate. $x = 4$ mm.

Readings.	Half differences.	$a - x$.	b.	Percentage of light transmitted.	Percentage of light transmitted. Mean of A and B.
A.					
cm.	cm.				
96·2–106·3	5·0	196·6	207·0	90·20	
96·3–106·4	5·0	196·6	207·0	90·20	
96·3–106·4	5·0	196·6	207·0	90·20	
96·4–106·5	5·0	196·6	207·0	90·20	
				90·20	
B.					
94·1–104·9	5·4	196·2	207·4	89·49	
94·4–104·6	5·1	196·5	207·1	90·02	
94·4–105·1	5·3	196·3	207·3	89·67	
95·3–104·9	4·8	196·8	206·8	90·56	
				89·93	90·07

Crown Glass. 15 mm. plate. $x = 5$ mm.

Readings.	Half differences.	$a - x$.	b.	Percentage of light transmitted.	Percentage of light transmitted. Mean of A and B.
A.					
cm.	cm.				
95·5–107·1	5·8	195·7	207·8	88·69	
95·8–106·9	5·5	196·0	207·5	89·22	
95·6–106·6	5·5	196·0	207·5	89·22	
				89·04	
B.					
94·3–104·9	5·3	196·2	207·3	89·58	
94·0–105·2	5·6	195·9	207·6	89·04	
94·2–105·4	5·6	195·9	207·6	89·04	
				89·22	89·13

Crown Glass. 18·5 mm. plate. $x = 6$ mm.

Readings.	Half differences.	$a - x$.	b.	Percentage of light transmitted.	Percentage of light transmitted. Mean of A and B.
A.					
cm.	cm.				
97·1–108·7	5·8	195·6	207·8	88·60	
96·7–109·0	6·1	195·3	208·1	88·08	
96·2–108·1	5·9	195·5	207·9	88·43	
				88·37	
B.					
94·7–105·8	5·5	195·9	207·5	89·13	
94·6–106·4	5·9	195·5	207·9	88·43	
94·6–106·4	5·9	195·5	207·9	88·43	
				88·66	88·51

Crown Glass. 24·3 mm. plate. $x = 8$ mm.

Readings.	Half differences.	$a - x$.	b.	Percentage of light transmitted.	Percentage of light transmitted. Mean of A and B.
A.					
cm.	cm.				
96·6–110·7	7·0	194·2	209·0	86·34	
96·4–109·9	6·7	194·5	208·7	86·86	
96·2–109·8	6·8	194·4	208·8	86·68	
				86·63	
B.					
94·1–106·7	6·3	194·9	208·3	87·55	
94·0–106·3	6·1	195·1	208·1	87·90	
94·4–107·0	6·3	194·9	208·3	87·55	
				87·70	87·16

Flint Glass. 7 mm. plate. $x = 3$ mm.

Readings.	Half differences.	$a - x$.	b.	Percentage of light transmitted.	Percentage of light transmitted. Mean of A and B.
A.					
cm.	cm.				
95·8–108·4	6·3	195·4	208·3	88·00	
96·2–108·1	6·0	195·7	208·0	88·52	
96·1–107·1	5·5	196·2	207·5	89·40	
96·5–108·1	5·8	195·9	207·8	88·87	
				88·70	
B.					
93·5–105·0	5·7	196·0	207·7	89·05	
94·0–105·4	5·7	196·0	207·7	89·05	
94·2–105·5	5·7	196·0	207·7	89·05	
94·3–106·1	5·9	195·8	207·9	88·70	
				88·94	88·83

Flint Glass. 49 mm. thick. $x = 19$ mm.

Readings.	Half differences.	$a - x$.	b.	Percentage of light transmitted.	Percentage of light transmitted. Mean of A and B.
A.					
cm.	cm.				
94·5–109·2	7·3	192·8	209·3	84·85	
94·5–109·0	7·2	192·9	209·2	85·02	
94·9–109·0	7·0	193·1	209·0	85·36	
95·3–109·2	6·9	193·2	208·9	85·53	
				85·19	
B.					
93·1–106·8	6·8	193·3	208·8	85·70	
93·3–107·4	7·0	193·1	209·0	85·36	
93·1–107·0	6·9	193·2	208·9	85·53	
93·9–107·3	6·7	193·4	208·7	85·88	
				85·62	85·40

Flint Glass. 69·5 mm. thick. $x = 27$ mm.

Readings.	Half differences.	$a - x$.	b.	Percentage of light transmitted.	Percentage of light transmitted. Mean of A and B.
A.					
cm.	cm.				
93·1–110·3	8·6	190·7	210·6	81·99	
93·5–110·3	8·4	190·9	210·4	82·32	
93·6–110·4	8·4	190·9	210·4	82·32	
				82·21	
B.					
92·5–108·9	8·2	191·1	210·2	82·65	
92·9–108·9	8·0	191·3	210·0	82·98	
93·1–109·0	7·9	191·4	209·9	83·15	
				82·93	82·57

Flint Glass. 91·3 mm. thick. $x = 35$ mm.

Readings.	Half differences.	$a - x$.	b.	Percentage of light transmitted.	Percentage of light transmitted. Mean of A and B.
A.					
cm.	cm.				
92·7–110·6	8·9	189·6	210·9	80·82	
92·9–110·9	9·0	189·5	211·0	80·66	
93·1–110·7	8·8	189·7	210·8	80·98	
				80·82	
B.					
91·6–110·1	9·2	189·3	211·2	80·34	
92·0–110·2	9·1	189·4	211·1	80·50	
92·3–109·8	8·7	189·8	210·7	81·15	
				80·66	80·74

Three plates of 6 mm. Crown Glass. Cemented together. $x = 6$ mm.

Readings.	Half differences.	$a - x$.	b.	Percentage of light transmitted.
A.				
cm.	cm.			
96·2–108·6	6·2	195·2	208·2	87·90
96·3–108·4	6·0	195·4	208·0	88·25
96·3–108·6	6·1	195·3	208·1	88·08
				88·08

Ordinary Plate Glass. 6 mm. thick.

Readings.	Half differences.	$a - x$.	b.	Percentage of light transmitted.	Percentage of light transmitted. Mean of A and B.
A.					
cm.	cm.				
94·9–108·4	6·7	195·1	208·7	87·39	
94·9–108·4	6·7	195·1	208·7	87·39	
94·9–108·6	6·8	195·0	208·8	87·22	
				87·33	
B.					
94·4–107·3	6·4	195·4	208·4	87·91	
94·4–107·0	6·3	195·7	208·3	87·91	
94·1–106·9	6·4	195·4	208·4	88·27	
				88·02	87·68

These determinations were carefully made, and the results are fairly concordant; as, however, this agreement was not inconsistent with the existence of a constant source of error, it appeared desirable to repeat some of them by a different method.

A polarising photometer was, therefore, set up (Plate 8, fig. 2), consisting of two Nicols and a right-angled prism; the Nicols, which were furnished with divided circles and verniers reading to 1°, were placed in the same straight line, and about 30 cm. apart, the right-angled prism being between them, and so placed that the field of view of the analysing Nicol was bisected vertically by the edge of the prism.

Two pieces of white paper were fixed in vertical planes at right angles to one another, both being illuminated by a small Argand gas burner; one was seen directly through the two Nicols, and the other, through the analyser only, by reflection in the prism. A blackened diaphragm was fixed between the prism and the second piece of paper, the aperture being of the same apparent size as the circular diaphragm of the polarising Nicol.

On looking through the analysing Nicol a circular white field was seen, bisected vertically, the two halves being usually of unequal brightness. On rotating the analyser one half of the field (that due to the reflection of the second paper in the prism), remained unchanged, whilst the other varied in brightness, being quite dark in two positions (when the Nicols were "crossed"). There were, of course, four positions of the analyser in which the two halves of the field appeared equally illuminated.

The light reflected by the right-angled prism was examined with a double-image prism and a plate of selenite, and was found to be completely unpolarised.

The observations were made by first determining the position of the analyser in which the field appeared equally bright throughout; then the plate of glass to be examined was placed between the right-angled prism and the white surface, and the new position of the analyser, in which there was equality of illumination, observed. As the intensity of the light which traversed the two Nicols varied as the square of the cosine of the angle between their principal sections, the percentage amount transmitted by the glass was given by $100 \times \cos^2 \alpha' / \cos^2 \alpha$, where α is the angle between the principal sections of the Nicol, when the field was uniformly bright without the glass, and α' when it was interposed.

The analysing Nicol was first rotated "clockwise," and readings made in each of the four quadrants of the position in which the two halves of the field appeared equally illuminated; the Nicol was then rotated "counter-clockwise," and four similar readings made, the mean of the eight readings being taken as the true position. The glass was then interposed between the diaphragm and the reflecting prism, and eight readings of the new position of the analyser, in which there was equality, made in the same way.

To determine the light transmitted by each piece of glass, four sets of eight observations were made without the glass, and four sets with it. Table III. gives the first set of each for the 6·5 mm. plate of crown glass; the other sets were about as concordant.

TABLE III.

Without Glass.

Readings of analyser.		Mean.*	α.
42° 25′	42° 45′	42° 35′	47° 25′
137 0	139 30	138 15	48 15
222 25	223 15	222 50	47 10
317 35	318 30	318 0	48 0
		Mean . . .	47 42

* As the intensity of the light varies as the square of the cosine, the geometrical, and not the arithmetical, is the true mean; but the observations did not appear to be sufficiently concordant to make it worth while to employ the longer process.

With 6·5 mm. plate of Crown Glass.

Readings of analyser.		Mean.*	α.
40° 15′	38° 0′	39° 07′	50° 53′
138 30	139 05	138 47	48 47
222 0	223 45	222 52	47 08
319 30	320 20	319 55	49 55
		Mean . . .	49 11

Table IV. gives the values of α determined in this way for the 6·5 mm., 11·5 mm., and 15 mm. plates of crown glass, and the percentage amount of light transmitted, as calculated from these numbers.

Table IV.

Crown glass. 6·5 mm. plate.

α.		Percentage amount of light transmitted.
Without glass.	With glass.	
47° 42′	49° 11′	94·32
45 44	49 36	86·22
46 16	48 37	91·45
46 54	48 37	93·61
		Mean 91·40

11·5 mm. plate.

α.		Percentage amount of light transmitted.
Without glass.	With glass.	
42° 03′	44° 53′	91·05
40 14	43 45	89·54
41 16	44 37	89·68
40 09	42 54	91·85
		Mean 90·53

15 mm. plate.

α.		Percentage amount of light transmitted.
Without glass.	With glass.	
41° 54′	45° 29′	88·73
40 23	43 42	90·08
40 17	43 44	89·82
		Mean 89·54

This method is clearly incapable of giving very accurate results. It is difficult to judge of the equality of the illumination in the two halves of the field, and also the angle through which the Nicol has to be turned to make the comparison is small, and, therefore, a slight error in the determination of the value of α makes a very considerable one in the result.

The measurements with the polarising photometer not being entirely satisfactory, another form of photometer was devised (Plate 8, figs. 3 and 3A). It consisted essentially of two white surfaces illuminated by the same lamp, the light falling very nearly perpendicularly on both. One surface was at a constant distance from the lamp, whilst the other could be brought nearer to, and moved further from it ; one surface was seen directly, and the other through the glass to be examined, and then the distance of the movable surface from the lamp altered till both surfaces appeared equally bright.

One surface was fixed at a distance of 78 cm. from the lamp and in the same horizontal plane, whilst the second was fixed to a vertical screen, which could be moved backwards and forwards by means of a pulley and catgut band along a board with a divided scale. The surfaces consisted of a double thickness of white paper, as it was found by taking a double thickness the apparent illumination was increased, a portion of the light which passed through the first paper being reflected back by the second. In front of the lamp two right-angled prisms were placed. They were held in position by two pieces of wood, through which a screw was passed, one prism being slightly in advance of the other, and overlapping it to a small extent. To adjust the prisms they were placed on a smooth table, the one resting directly on the table, and the other on a thin piece of card, and, after being adjusted, fixed in position by means of the screw. When so placed, the line dividing the two fields of view was much narrower than when the front surfaces of the prisms were in the same vertical plane and their edges in contact. A screen was placed between the prisms and the lamp, which were only 8 cm. apart.

The light of the lamp fell nearly perpendicularly on the white paper, and the direction of that reflected back to the prism was also nearly normal to the paper, so that the distance of the paper from the lamp could be altered without altering to any considerable extent the angle at which the light fell upon it, or the angle under which it was seen.

The distance of the moveable surface from the lamp, when its reflection in the prism and that of the fixed surface appeared equally bright, was determined (1) without any glass being interposed, (2) with a plate of glass between the fixed surface and the prisms, and (3) between the moveable surface and the prisms, six readings being made of each of these positions, and then the prisms reversed so that the surface which had been seen by reflection in the one was seen by reflection in the other, and six more readings made.

Calling the apparent brightness of the fixed surface C, the distance of the moveable surface from the lamp without the glass x, with the glass between the fixed surface and the prisms x', and with the glass between the moveable surface and the prisms x'', and the coefficient of transparency of the particular piece of glass k, then the two surfaces will appear equally bright when

$$C = \frac{1}{x^2}; \qquad Ck = \frac{1}{(x')^2}; \qquad \text{and } C = k\frac{1}{(x'')^2};$$

whence

$$k = \left(\frac{x}{x'}\right)^2, \quad \text{or} \quad \left(\frac{x''}{x}\right)^2, \quad \text{or more simply} \quad \left(\frac{x''}{x'}\right).$$

The percentage amount of light transmitted by the 6·5 mm. and the 24·3 mm. plates of crown glass was determined in this way. Table V. gives the results.

TABLE V.

6·5 mm. plate.

Readings.		Percentage of light transmitted.
x''.	x'.	
cm.	cm.	
78·0	84·4	92·42
77·8	84·9	91·64
		Mean 92·03

24·3 mm. plate.

Readings.		Percentage of light transmitted.
x''.	x'.	
cm.	cm.	
78·0	87·0	89·65
74·1	85·9	86·26
		Mean 87·95

In order to have obtained really accurate results with this method it would have been necessary to have made a large number of observations, and taken their mean; but, as the results obtained with it, and with the polarising photometer, agreed fairly well with one another and with the far larger number of observations made with the first method, it was thought unnecessary to continue the observations, the agreement between those obtained by all three methods being sufficient to show that there was but slight, if any, probability of a constant error due to the photometer itself used in the first series.

Table VI. gives the results of the three methods and the probable error of each determination calculated by the ordinary formula, $0·674\sqrt{(\Sigma e^2)}\,.\,/n$.

TABLE VI.—Percentage Amount of Light Transmitted.

	First method.	Second method.	Third method.
Crown glass—			
6·5 mm. plate	91·50 ± ·06	91·40 ± 1·07	92·03 ± ·18
11·5 ,, ...	90·07 ,, ·07	90·53 ,, ·32	
15·0 ,, ...	89·13 ,, ·07	89·54 ,, ·22	
18·5 ,, ...	88·51 ,, ·08		
24·3 ,, ...	87·16 ,, ·15	. .	87·95 ,, ·81
Flint glass—			
7·0 mm. thick	88·83 ,, ·09		
49·0 ,, ...	85·40 ,, ·07		
69·5 ,, ...	82·57 ,, ·11		
91·3 ,, ...	80·74 ,, ·07		
Common plate glass—			
6 mm. thick	87·68 ,, ·10		
3 plates of 6 mm. crown glass .	88·08 ,, ·05		

Section II.—Amount of Light Reflected at a nearly Perpendicular Incidence.

The small percentage of light reflected by glass at a perpendicular incidence rendered its direct determination difficult; a fairly satisfactory method of measurement was, however, at length devised. The principle of the method was the obvious one of comparing the amount of light which reached the photometer when it came direct from the lamp with that which reached it after reflection from the glass.

It was found necessary to use two lamps, as no single-lamp apparatus, such as had been used for the transmission experiments, could be employed.

One of the lamps, a small Argand gas burner, was placed at the end of the photometer board (Plate 8, fig. 1), and a similar gas burner attached to the arm of a goniometer fixed at a short distance from the other end of the photometer board, the vertical axis of the goniometer being in the prolongation of the median line of the board. The glass of which the reflective power was to be determined could be placed with its surface vertical, and in the axis of the goniometer.

The experiments were made by first comparing the illumination produced by the two lamps when the light from both fell directly on the "photometer," the arm of the goniometer carrying the lamp being in the prolongation of the line joining the fixed lamp and the axis of the goniometer; the glass plate was then attached to the goniometer with its surface vertical and nearly normal to the line joining the fixed lamp and the axis of the goniometer, the arm of the goniometer rotated until the light again fell on the photometer after reflection from the glass plate, and the position of equality determined.

It was not necessary that the illuminating power of the two lamps should be equal: it was necessary that the ratio between their illuminating powers should remain as nearly constant as possible. The gas for the two burners came from the same supply pipe, and was passed through a bell-and-valve regulator; in spite of the regulator, it was found impossible to get satisfactory measurements, except in fairly still weather; the slight flickerings in the flames which occurred whenever there was much wind prevented the position of equality of illumination being determined with any degree of accuracy.

The measurements were always made in a dark room, a "detector" gas burner being used to read the position of the index on the scale; and, in order to reduce the stray light as much as possible, cylindrical metal chimneys, 5·5 cm. in diameter and 18 cm. in height, blackened externally and internally, were placed round the glass chimneys of the burners, a rectangular aperture being cut in each at the level of the flame. Two black wood screens with square openings, similar to those used in the transmission experiments (p. 248), were fixed at either end of the photometer board; there were black cloth screens behind and above the board, and the walls and ceiling of the room were painted a dead black; the metal clamp by which the glass plate was attached to the goniometer was made as small as possible and blackened, and the goniometer itself was covered with a black cloth whilst the observations were being made.

In order to measure the intensity of the light reflected by the glass both when incident normally and at various angles, that is, to compare the intensity of light which under certain circumstances would be partially polarised with the intensity of unpolarised light, it was necessary that the photometric surface should be normal both to the incident light and to the line of sight; hence, the photometer which had been used in the transmission experiments clearly could not be used, nor indeed could a Bunsen's disk or any of its modifications. A new form of photometer was, therefore, devised. Two wooden screens were fixed to the sides of a block 10 cm. across, similar to the one which had carried the photometer in the first set of experiments; in these rectangular apertures were cut, 3 cm. by 2 cm., and "parchment" paper fastened over them; the two right-angled glass prisms which had been used in the third method for determining the amount of the transmitted light (Plate 8, fig. 3A, p. 260) were placed between the screens, and in a line with the apertures. The two papers, each illuminated by the light of one of the lamps, were seen by reflection in the prisms, and by moving the block, to which an index was fixed, along the photometer board a position could be found in which the two images appeared equally bright.

Glass only reflecting from 4 to 5 per cent. of the light incident normally upon its surface, and the photometer scale being only 2 metres long, it was impossible to compare the intensities of the direct and reflected light when the two lamps had the same illuminating power. During the first set of experiments, those marked A, the necessary difference was obtained by keeping the flame of the comparison lamp turned down rather low. Subsequently, in the determinations marked B, the same result was obtained by different sized apertures in the metal chimneys; that in the chimney of the goniometer lamp was 10 mm. by 18 mm., whilst the one in the chimney of the comparison or fixed lamp was only 10 mm. by 6 mm.; the gas flames were so regulated that the apertures appeared completely filled with a uniformly bright flame. When the gas pressure changes slightly, the size of a flame, and not its intrinsic brilliancy, is mainly what alters; and, therefore, by limiting the visible portion of the flame in this way a greater constancy in the ratio between the illumination produced by the two lamps was obtained; but, as the tables show, the measurements made when the whole flame, and those in which the central portions only were used, agree satisfactorily.

This method for determining the reflective power possesses the obvious defect that, owing to the necessary alteration in the course of the light, the direct and reflected light cannot be interchanged, and, therefore, a constant source of error may easily exist. It seems probable that, to a small extent, such was the case, and that the measured amounts of the reflected light were slightly too high.

When the lamp attached to the goniometer was so placed that the light fell directly on the photometer, the whole beam of light which passed through the aperture in the chimney fell solely on the screen at the end of the photometer board, and no light diffused from the lamp reached the photometer. When, however, the lamp was so placed that the light was reflected from the glass, part of the light fell on the wall of the room, and, although this was at a distance of about 2 metres and painted black, some light must have been diffused from it towards the photometer; the metal clamp also in which the glass was held, although made as small as possible and blackened, certainly reflected some light. In order to obtain some idea of the amount of light which reached the photometer from these sources, the clamp was fixed to the goniometer without any glass; the left half of the field was not absolutely dark, but the amount of illumination was far too slight to be measurable, in fact it was almost imperceptible when the light from the comparison lamp illuminated the other half of the field; hence, the error from this cause can only be small.

In order to eliminate as far as possible any error due to a change in the relative amount of illumination produced by the lamps, four readings were first made of the position of the photometer when the two halves of the field were equally bright, the light of both lamps falling directly on it; four readings of the position when the light was reflected from the glass were then made; and then four more with the direct light: the mean of the eight readings being taken as the true position of the photometer when the light reached it directly.

Calling the two sources of light m and n, the distance between them x, the two positions of the photometer in which there is equality of illumination x_1 and x_2, and K the coefficient of reflection for the particular plate of glass, then

$$\frac{m}{x_1^2} = \frac{n}{(x - x_1)^2} \quad \text{and} \quad \frac{\mathrm{K}m}{x_2^2} = \frac{n}{(x - x_2)^2},$$

$$m = \frac{nx_1^2}{(x - x_1)^2} \quad \text{and} \quad \mathrm{K}m = \frac{nx_2^2}{(x - x_2)^2};$$

therefore

$$\mathrm{K} = \frac{x_2^2 (x - x_1)^2}{x_1^2 (x - x_2)^2} \quad \text{or} \quad \left\{\frac{x_2 (x - x_1)}{x_1 (x - x_2)}\right\}^2.$$

The lamps not being placed at the ends of the divided scale and the two translucent screens of the photometer being necessarily at some distance apart, in order to obtain x_1 and x_2 the distance between the lamp and the zero of the scale had to be added, and the distance between the translucent screen and the index subtracted from the scale reading; thus, the distance from the axis of the lamp carried by the goniometer to the zero of the scale being 45·6 cm., in the first series of experiments, and the index of the photometer being at a distance of 5·7 cm. from the paper, the scale readings, plus 39·9, gave the value of x_1 and x_2. Similarly, 16·9 cm. added to the difference between the readings and 200 gave the value of $(x - x_1)$ and $(x - x_2)$.

In the first set of experiments the distances from the lamps were measured from the axes of the flames, which were about 15 mm. in diameter; in all the others, for which portions only of the flames were used, the distances were measured from the apertures in the chimneys.

The same glass was used for these experiments that had been employed in the transmission experiments already described. In order to prevent the light reflected from the second surface of the glass reaching the photometer the measurements were made with prisms cut from the 18·5 mm. plate of crown glass, and from the flint glass block; one face of each of these prisms was formed by one of the original surfaces of the glass, and these faces, which had not been repolished, were used for the reflection experiments.

Table VII. gives some of the actual measurements made with the prism of crown glass. The first four columns contain measurements made with the light from the whole surface of the flames; the others with light from the central portions only. In the first set the light was incident upon the glass at an angle of 6° 17′, and in the second at 6° 47′.

TABLE VII.

A.				B.			
Direct light.	Mean with direct light.	Reflected light.	Mean with reflected light.	Direct light.	Mean with direct light.	Reflected light.	Mean with reflected light.
cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.
128·8				117·0			
129·8				118·9			
127·8		28·8		118·1		24·0	
130·1		30·1		119·3		24·2	
	128·8	28·0	28·7		118·9	23·8	24·1
127·8		28·0		118·6		24·3	
129·4				120·1			
127·6		29·2		119·3		23·7	
129·1		28·7		120·2		24·2	
	127·9	27·7	28·8		119·4	24·1	24·2
126·3		29·6		118·3		24·7	
128·0				120·3			
126·7		28·5		119·2		24·1	
128·1		28·7		119·6		25·0	
	127·2	28·0	28·4		118·8	24·5	24·4
126·1		28·6		118·3		24·2	
128·0				118·1			
126·3		27·8		118·2		23·5	
127·9		28·9		118·6		24·3	
	127·0	27·6	28·2		118·3	24·2	24·1
126·9		28·4		118·1		24·3	
128·0				117·5			
125·8				118·5			
127·3				119·6			

From the mean results the values of x_1, x_2, $(x - x_1)$ and $(x - x_2)$ were obtained, and from these the percentage amount of light reflected by the glass calculated by the formula $K = \left\{\frac{x_2(x - x_1)}{x_1(x - x_2)}\right\}^2$. These values, and others obtained from similar sets of measurements, are contained in Table VIII.

TABLE VIII.

Crown Glass.

Angle of incidence.	x_1.	$(x - x_1)$.	x_2.	$(x - x_2)$.	Per cent. of incident light reflected.
A. 6° 17′	168·7	88·1	68·6	188·2	3·62
	167·8	89·0	68·7	188·1	3·75
	167·1	89·7	68·3	188·5	3·78
	166·9	89·9	68·1	188·7	3·78
6° 17′	171·5	85·3	71·7	185·1	3·71
	173·6	83·2	74·5	182·3	3·84
	174·3	82·5	74·0	182·8	3·67
	175·6	81·2	76·4	180·4	3·83
B. 6° 47′	156·1	95·2	61·3	190·0	3·87
	156·6	94·7	61·4	189·9	3·82
	156·0	95·3	61·6	189·7	3·93
	155·5	95·8	61·3	190·0	3·95
7° 30′	158·4	92·5	62·2	188·7	3·70
	157·8	93·1	61·7	189·2	3·70
	157·8	93·1	62·4	188·5	3·81
	157·7	93·2	62·3	188·6	3·81
	157·5	93·4	61·8	189·1	3·76
				Mean	3·78

Similar measurements were made with the flint glass wedge. The results are given in Table IX.

TABLE IX.

Flint Glass.

Angle of incidence.	x_1.	$(x - x_1)$.	x_2.	$(x - x_2)$.	Per cent. of incident light reflected.
10°	156·2	94·7	68·2	182·7	5·12
	155·9	95·0	67·8	183·1	5·09
	155·1	95 8	68·7	182·2	5·42
	155·6	95·3	69·1	181·8	5·42
10°	157·6	93·3	68·9	182·0	5·02
	157·6	93·3	69·4	181·5	5·12
	157·2	93·7	69·4	181·5	5·20
				Mean	5·20

Section III.—Amount of Light Reflected at a nearly Perpendicular Incidence after Repolishing.

Lord RAYLEIGH found ('Roy. Soc. Proc.,' vol. 41, p. 389) that, although the glass surfaces he examined were free from any apparent tarnish, the amount of light they reflected was largely increased by repolishing. The wedge of crown glass was, therefore, repolished on December 21, 1887, by means of a disk of wood charged with putty powder and mounted in a lathe (the same method that Lord RAYLEIGH had used), and its reflective power redetermined immediately; it was found to reflect 4·29, instead of 3·78, per cent. The glass was again examined on January 5, 1888, and it then reflected 4·20 per cent.

Two days later the glass was repolished a second time with fine rouge, and again examined; it reflected 4·22 per cent. of the incident light.

After an interval of five months, on June 13, this piece of glass, the surface of which had become considerably tarnished, was rubbed with wash-leather until the moisture deposited on the glass by breathing gently on it evaporated quite uniformly; it was then examined, and found to reflect 4·42 per cent. of the incident light. The next day it was repolished for the third time, and examined immediately; it reflected 4·30 per cent.

An attempt was made to repolish the flint glass wedge on February 28, 1888, with both putty and rouge, but a surface free from scratches could not be obtained. Its reflective power, however, was increased, and it reflected 6·20 per cent., instead of 5·20, after this imperfect polishing.

The surface not being satisfactory, the glass was sent to Mr. HILGER to be repolished. It was received back on the evening of March 2, and examined on the 3rd; it reflected 6·06 per cent.

After three months the glass was again examined, the film which had formed on its surface having been previously removed by rubbing with a wash-leather; it only reflected 5·71 per cent., although the surface appeared perfectly polished. On June 11 it was repolished with very fine washed rouge, and was found to reflect 6·25 per cent. of the incident light. Two days later, on June 13, it only reflected 5·73 per cent.

The polishing with putty powder was effected by means of a soft wood disk, mounted in a lathe, the disk being kept moist. For the rouge a polisher was formed by cementing a piece of silk to a sheet of plate glass fastened to a table, and charging this with carefully washed rouge: the glass was held against the rapidly rotating disk in the one case, and rubbed over the fixed surface in the other. The relative velocities of glass and polisher were very different in the two cases, and with the disk the friction was sufficiently great for the glass to become sensibly warm.

Tables X. and XI. give the details of these experiments; with the crown glass the angle of incidence was 7° 30′, and with the flint glass 10°.

TABLE X.

Crown Glass.

	x_1.	$(x - x_1.)$	x_2.	$(x - x_2.)$	Per cent. of incident light reflected.
December 21, 1887.—Repolished with putty	157·8	93·1	65·0	185·9	4·25
	157·0	93·9	64·8	186·1	4·34
				Mean	4·29
January 5, 1888	157·8	93·1	64·3	186·6	4·13
	156·6	94·3	63·9	187·0	4 23
	156·0	94·9	63·5	187·4	4·25
				Mean	4·20
January 7, 1888.—Repolished with rouge	155·7	95·2	63·0	187·9	4·20
	154·9	96·0	62·6	188·3	4·24
	155·5	95·4	63·3	187·6	4·28
	156·2	94·7	63·2	187·7	4·17
				Mean	4·22
June 13	159·6	91·3	67·5	183·4	4·43
	159·6	91·3	67·3	183·6	4·40
	159·8	91·1	67·7	183·2	4·44
				Mean	4·42
June 14.—Repolished with rouge	160·8	90·1	68·1	182·8	4·36
	160·9	90·0	67·6	183·3	4·25
	160·9	90·0	67·8	183·1	4·29
				Mean	4·30

TABLE XI.

Flint Glass.

	x_1.	$(x - x_1.)$	x_2.	$(x - x_2.)$	Per cent. of incident light reflected.
February 28, 1888.—Repolished with rouge and putty	154·6	96·3	71·2	179·7	6·09
	153·4	97·5	70·8	180·1	6·24
	152·9	98·0	70·5	180·4	6·27
				Mean	6·20
March 3, 1888, 5 P.M. — Repolished by Mr. HILGER.	156·7	94·2	73·6	177·3	6·23
	157·4	93·5	73·0	177·9	5·94
	157·4	93·5	73·8	177·1	6·13
	157·1	93·8	73·6	177·3	6·14
				Mean	6·11
March 3, 1888, 9 P.M.	156·1	94·8	71·6	179·3	5·88
	155·4	95·5	71·5	179·4	6·00
	155·4	95·5	71·6	179·3	6·02
	155·2	95·7	72·1	178·8	6·17
				Mean	6·02
June 5, 1888	159·9	91·0	74·1	176·8	5·69
	159·9	91·0	74·7	176·2	5·82
	160·2	90·7	74·9	176·0	5·82
	160·4	90·5	73·6	177·3	5·51
				Mean	5·71
June 7, 1888	160·2	90·7	75·7	175·2	5·98
	160·8	90·1	74·4	176·5	5·58
	160·6	90·3	75·3	175·6	5·81
	159·7	91·2	75·3	175·6	6·00
				Mean	5·84
June 11, 1888	160·5	90·4	74·0	176·9	5·55
	160·5	90·4	74·3	176·6	5·61
	160·5	90·4	74·6	176·3	5·68
	161·0	89·9	74·6	176·3	5·58
				Mean	5·60
June 11, 1888.—Repolished with rouge	166·0	84·9	81·5	169·4	6·05
	165·7	85·2	82·5	168·4	6·34
	165·8	85·1	82·6	168·3	6·35
				Mean	6·25
June 13, 1888	160·1	90·8	74·5	176·4	5·74
	160·4	90·5	74·4	176·5	5·66
	160·5	90·4	75·1	175·8	5·79
				Mean	5·73

Section IV.—Amount of Light Reflected at Various Incidences between 0° *and* 90° *by the Crown Glass before and after Repolishing.*

In the experiments already described the light incident on the glass consisted of a divergent beam, or rather of a complex of rays, the mean incidence of which, if such a term may be used, was as nearly normal as the construction of the apparatus permitted.

Measurements were also made with the crown glass both before and after it was repolished, when the light was incident upon its surface at various angles, the angle between the axis of the beam of light and the normal to the surface being considered as the angle of incidence. Table XII. gives these results.

TABLE XII.—Crown Glass.

Angle of incidence.	Before repolishing.					After repolishing.				
	x_1.	$(x-x_1)$.	x_2.	$(x-x_2)$.	Per cent. of light reflected.	x_1.	$(x-x_1)$.	x_2.	$(x-x_2)$.	Per cent. of light reflected.
10° B.	156·5	94·6	61·4	189·7	3·83	155·5	95·4	63·1	187·8	4·25
	156·7	94·4	61·5	189·6	3·82	150·0	95·9	63·1	187·6	4·36
	156·8	94·3	61·2	189·9	3·76	154·4	96·5	63·5	187·4	4·48
						154·0	96·9	63·6	187·3	4·56
				Mean	3·80				Mean	4·41
20° B.	157·2	93·9	61·1	190·0	3·69	154·9	96·0	66·1	184·8	4·91
	156·2	94·9	60·9	190·2	3·78	154·7	96·2	65·3	185·6	4·79
	155·9	95·2	60·7	190·4	3·79	154·7	96·2	65·1	185·8	4·75
	155·7	95·4	60·7	190·4	3·81	154·4	96·5	65·2	185·7	4·81
				Mean	3·77				Mean	4·81
30° A.	171·8	85·0	73·8	183·0	3·98					
	170·4	86·4	74·1	182·7	4·23					
	170·6	86·2	73·1	183·7	4·04					
	170·6	86·2	73·1	183·7	4·04					
B.	155·0	96·1	59·3	191·8	3·67	153·6	97·3	63·0	187·9	4·51
	154·4	96·7	59·9	191·2	3·86	152·9	98·0	62·8	188·1	4·58
	153·8	97·3	59·7	191·4	3·89	153·0	97·9	62·9	188·0	4·58
	154·2	96·9	58·7	192·7	3·68	152·8	98·1	62·5	188·4	4·53
				Mean	3·92				Mean	4·55
40° B.	156·4	94·7	64·7	186·4	4·42	158·3	92·6	72·0	178·9	5·54
	156·5	94·6	65·6	185·5	4·57	158·1	92·8	71·3	179·6	5·43
	156·3	94·8	65·8	185·3	4·64	158·2	92·7	70·4	180·5	5·22
	155·7	95·4	64·4	186·7	4·47	157·4	93·5	69·3	181·6	5·14
						156·6	94·3	68·9	182·0	5·19
				Mean	4·52				Mean	5·30

TABLE XII.—continued.

Angle of incidence.	Before repolishing.					After repolishing.				
	x_1.	$(x-x_1)$.	x_2.	$(x-x_2)$.	Per cent. of light reflected.	x_1.	$(x-x_1)$.	x_2.	$(x-x_2)$.	Per cent. of light reflected.
50° A.	162·2	94·6	72·5	184·3	5·26					
	161·3	95·5	71·9	184·9	5·30					
	161·2	95·6	72·2	184·6	5·38					
	161·2	95·6	72·4	184·4	5·42					
B.	154·3	96·8	68·6	182·5	5·56	158·6	92·3	75·6	175·3	6·19
	152·8	98·3	67·8	183·3	5·66	158·3	92·6	76·2	175·3	6·29
	152·4	98·7	68·3	182·8	5·85	158·3	92·6	74·7	176·2	6·46
	151·9	99·2	67·7	183·4	5·81	157·9	93·0	74·5	176·4	6·15
				Mean	5·53				Mean	6·27
56° 34′ B.	156·4	94·7	77·3	173·8	7·25	155·3	95·6	79·7	171·2	8·21
	158·7	92·4	78·4	172·7	6·99	155·5	95·4	79·9	171·0	8·22
	159·5	91·6	79·2	171·9	7·00	155·2	95·7	79·9	171·0	8·30
	157·7	93·4	79·9	171·2	7·64	154·9	96·0	79·4	171·5	8·23
				Mean	7·22				Mean	8·24
60° A.	163·1	93·7	85·6	171·2	8·25					
	163·2	93·6	84·9	171·9	8·02					
	161·9	94·9	85·3	171·5	8·50					
	161·8	95·0	84·1	172·7	8·17					
B.	154·4	96·7	80·2	170·9	8·64	155·1	95·8	84·5	166·4	9·84
	153·4	97·7	80·7	170·4	9·10	154·5	96·4	83·5	167·4	9·68
	152·2	98·9	79·5	171·6	8·88	153·9	97·0	83·7	167·2	9·95
	151·9	99·2	78·3	172·8	8·76	153·7	97·2	83·8	167·1	10·06
				Mean	8·54				Mean	9·88
65° A.	166·8	90·0	97·0	159·8	10·73					
	166·8	90·0	97·5	159·3	10·90					
	167·4	89·4	98·6	158·2	11·08					
	168·2	88·6	97·8	159·0	10·50					
B.	154·1	97·0	87·1	164·0	11·18	156·4	94·5	91·9	159·0	12·20
	153·3	97·8	87·1	164·0	11·48	155·8	95·1	91·5	159·4	12·28
	152·1	99·0	86·6	164·5	11·74	155·8	95·1	91·6	159·3	12·32
	151·5	99·6	85·9	165·2	11·66	155·4	95·5	91·2	159·7	12·31
				Mean	11·16				Mean	12·28
70° A.	167·9	88·9	109·8	147·0	15·64					
	167·6	89·2	109·8	147·0	15·80					
	168·0	88·8	110·0	146·8	15·69					
	169·5	87·3	111·6	145·2	15·67					

TABLE XII.—continued.

Angle of incidence.		Before repolishing.					After repolishing.				
		x_1.	$(x-x_1)$.	x_2.	$(x-x_2)$	Per cent. of light reflected.	x_1.	$(x-x_1)$.	x_2.	$(x-x_2)$.	Per cent. of light reflected.
70°	A.	169·2	87·6	109·1	147·7	14·62	158·7	92·2	105·8	145·1	17·94
		167·3	89·5	109·0	147·8	15·57	157·7	93·2	105·2	145·7	18·21
		166·0	90·8	106·8	150·0	15·17	157·0	93·9	104·4	146·5	18·16
		165·3	91·5	107·4	149·4	15·83	157·0	93·9	105·5	145·4	18·83
										Mean	18·28
	B.	156·9	94·2	99·3	151·8	15·42					
					Mean	15·49					
75°	B.						160·1	90·8	119·4	131·5	26·52
							159·5	91·4	118·9	132·0	26·64
							158·7	92·2	116·7	134·2	25·52
							157·8	93·1	117·3	133·6	26·83
										Mean	26·33

Section V.—Amount of Light Transmitted after Repolishing.

The effect of repolishing on the amount of light transmitted by the glass was examined. The first observations, those with the 24·3 mm. plate of crown glass (except the last) were made with the photometer used in the reflection experiments, the arm of the goniometer carrying the lamp being clamped in the prolongation of the line joining the fixed lamp and the axis of the goniometer. The small slot in front of the flame of the fixed lamp was subsequently replaced by one of the same size as that in front of the lamp carried by the goniometer, and most of the observations made with this arrangement, in which there were two lamps of nearly equal illuminating power. Finally, a photometer with a single lamp, two mirrors, and inclined paper surfaces, like that used for the original transmission experiments, was fitted up, and this was used for all measurements made in August and September, 1888, that is, those with the flint glass and the four last determinations of the light transmitted by the crown glass.

The measurements were made as described in Section I., but only four readings were taken in each position of the glass, and the results were calculated out by the expression $k = \frac{x_1(x - x_2)}{x_2(x - x_1)}$ (see p. 250), where k is the coefficient of transparency, x the distance between the two lights, and x_1 and x_2 the two positions of the photometer in which there is equality of illumination, the optical shortening of the path of the light due to its passage through the glass being of course allowed for.

The results are contained in Tables XIII. and XIV.

TABLE XIII.

Crown Glass.

	x_1.	$(x - x_1)$.	x_2.	$(x - x_2)$.	Per cent. of incident light transmitted.
6·5 mm. plate.					
July 6, 1888.—Cleaned, but not repolished .	127·2	135·1	133·1	129·2	91·39
	126·9	135·4	132·7	129·6	91·53
	127·7	134·6	133·4	128·9	91·67
				Mean	91·53
July 13.—"Ground grey and repolished" by Mr. HILGER	125·9	136·4	132·3	130·0	90·70
	127·2	135·1	133·3	129·0	91·11
	126·0	136·3	133·1	129·2	89·73
				Mean	90·31
July 17.—Re-examined	125·8	136·5	132·2	130·1	90·70
	126·0	136·3	132·0	130·3	91·25
	125·9	136·4	132·1	130·2	90·98
				Mean	90·90
11·5 mm. plate.					
August 13.—Cleaned, but not repolished .	187·8	196·8	198·0	186·6	89·93
	187·9	196·7	197·5	187·1	90·50
	187·8	196·8	197·5	187·1	90·40
				Mean	90·28
11·5 mm. plate *bis*.					
July 13.—"Ground grey and repolished" by Mr. HILGER	126·1	136·0	132·5	129·6	90·69
	126·1	136·0	133·6	128·5	89·18
	125·7	136·4	132·4	129·7	90·28
				Mean	90·05
July 16.—Re-examined	125·6	136·5	132·8	129·3	89·59
	125·6	136·5	133·0	129·1	89·32
	125·5	136·6	132·9	129·5	89·52
				Mean	89·48
July 17.—Re-examined	124·9	137·2	131·7	130·4	90·14
	124·5	137·6	132·0	130·1	89·18
	124·8	137·3	131·7	130·4	90·00
				Mean	89·77

TABLE XIII.—continued.

	x_1.	$(x - x_1)$.	x_2.	$(x - x_2.)$	Per cent. of incident light transmitted.
11·5 mm. plate *bis* (continued).					
August 13.—Repolished with fine rouge on silk, and subsequently (11/8/88) with putty powder on wood disk	187·4	197·2	198·2	186·4	89·37
	187·9	196·7	198·2	186·4	89·84
	188·7	196·9	198·3	186·3	90·03
				Mean	89·75
15 mm. plate.					
July 6.—Cleaned, but not repolished . . .	125·6	136·4	133·2	128·8	89·04
	126·1	135·9	133·6	128·4	89·18
	126·2	135·8	133·3	128·7	89·72
				Mean	89·31
July 13.—" Ground grey and repolished " by Mr. HILGER	125·3	136·7	134·1	127·9	87·42
	125·6	136·4	133·6	128·4	88·50
	126·1	135·9	133·9	128·1	88·77
				Mean	88·23
July 17.—Re-examined	124·8	137·2	132·2	129·8	89·31
	124·8	137·2	132·5	129·5	88·90
	124·9	137·1	132·7	129·3	88·77
				Mean	88·99
August 17.—Repolished with putty powder on wood disk	185·4	199·1	197·1	187·4	88·54
	185·5	199·0	197·0	187·5	88·72
	185·6	198·9	196·9	187·6	88·90
				Mean	88·72
18·5 mm. plate.					
July 16.—Cleaned, but not repolished . .	125·4	136·5	133·3	128·6	88·63
	124·9	137·0	133·1	128·8	88·22
				Mean	88·41
July 17.—Re-examined	123·5	138·4	131·6	130·3	88·35
	123·6	138·3	131·6	130·3	88·49
	124·4	137·5	131·8	130·7	89·72
				Mean	88·85
August 17.—Re-examined	184·6	199·8	197·2	187·2	87·71
	185·7	198·7	196·5	187·9	89·37
	185·2	199·2	197·4	187·0	88·07
				Mean	88·38

TABLE XIII.—continued.

	x_1.	$(x - x_1)$.	x_2.	$(x - x_2)$.	Per cent. of incident light transmitted.
24·3 mm. plate.					
June 15.—Cleaned, but not repolished . .	84·7	165·4	92·4	157·7	87·40
	84·8	165·3	92·5	157·6	87·41
	84·9	165·2	93·9	156·2	85·49
				Mean	86·90
June 16.—Re-examined	86·7	163·4	95·6	154·5	85·75
	87·7	162·4	95·3	154·8	87·72
	88·0	162·1	95·9	154·2	87·29
				Mean	86·92
June 16.—Polished with rouge on silk . . .	84·8	165·3	92·9	157·2	86·81
	85·5	164·6	93·3	156·8	87·30
	85·6	164·5	93·8	156·3	86·71
				Mean	86·93
June 23.—Repolished by Mr. HILGER; the polish was defective	84·6	165·5	94·6	155·5	84·03
	84·4	165·7	94·3	155·8	84·15
	84·2	165·9	94·3	155·8	83·85
	84·1	166·0	94·2	155·9	83·85
				Mean	83·97
June 30.—Repolished a second time by Mr. HILGER	85·5	164·6	92·9	157·2	87·90
	85·6	164·5	92·6	157·5	88·50
	84·6	165·5	92·5	157·6	87·09
				Mean	87·84
June 30.—Re-examined	86·4	163·3	94·0	156·1	87·86
	86·3	163·8	93·7	156·4	87·94
	86·7	163·4	94·3	155·8	87·67
				Mean	87·82
July 17.—Re-examined	123·0	138·7	132·0	129·7	87·14
	123·7	138·0	132·4	129·8	87·88
	123·3	138·4	131·7	130·0	87·94
				Mean	87·65

TABLE XIV.

Flint Glass.

	x_1.	$(x - x_1)$.	x_2.	$(x - x_2)$.	Per cent. of incident light transmitted.
7 mm. thick.					
September 30, 1888.—Cleaned, but not repolished	187·6	197·1	194·8	189·9	92·78
	187·7	197·0	194·7	190·0	92·98
	187·5	197·2	195·1	189·6	92·40
	187·5	197·2	195·0	189·7	92·50
				Mean	92·66
49 mm. thick.					
August 15.—Surface repolished with putty powder on August 14	183·0	200·1	198·4	184·7	85·14
	182·5	200·6	197·8	185·3	85·23
	182 9	200·2	198·1	185·0	85·32
	183·1	200·0	198·5	184·6	85·14
				Mean	85·21
August 16.—Re-examined	182·8	200·3	198·7	184·4	84·70
	183·2	199·9	198·0	185·1	85·68
	182·7	200·4	198·1	185·0	85·15
				Mean	85·18
August 16.—Repolished with putty powder and examined immediately	183·1	200·0	198·1	185·0	85·50
	182·3	200·8	198·9	184·2	84·08
	182·5	200·6	199·1	184·0	84·08
				Mean	84·55
August 18.—Re-examined	183·0	200·1	198·8	184·3	84·78
	182·7	200·4	198·7	184·4	84·61
	182·7	200·4	198·6	184·5	84·70
				Mean	84·70
69·5 mm. thick.					
August 15.—Cleaned, but not repolished .	182·1	200·2	198·5	183·8	84·22
	181·9	200·4	198·4	183·9	84·13
	182·1	200·2	198·0	184·3	84·67
	182·4	199·9	198·6	183·7	84·40
				Mean	84·35

Section VI.—Values of the Polarising Angles before and after Repolishing.

In order to determine the angles of polarisation, a form of apparatus essentially similar to that employed by SEEBECK ('POGGENDORFF, Annalen,' vol. 20, 1830, p. 27) was used. It consisted of a goniometer with a horizontal circle reading to 20″; the slit and lens of the collimator were removed and the observing telescope replaced by a tube to one end of which a vertical divided circle was fixed. A Nicol was contained in an inner tube, and by means of a vernier the position of its principal section could be read on the vertical circle to 5′. The stage of the goniometer and the arm carrying the Nicol were geared together by means of toothed wheels working into a double pinion, the number of teeth in the wheels and pinion being such that on moving the arm of the goniometer carrying the Nicol the angular velocity of the tube was twice that of the stage.

A small gas flame was placed close to the end of the collimator tube, the flame being surrounded by a blackened metal chimney with a small aperture in it, and the glass surface whose angle of polarisation was to be observed clamped to the vertical stage with its surface in the prolongation of the vertical axis of the goniometer, the stage turned till the image of the flame was seen through the Nicol, and then, by means of the pinion (the axis of which was fixed to a sliding piece), the stage and Nicol geared together.

The Nicol having been clamped with its short diagonal horizontal, the arm was moved till the light reflected by the glass was reduced to a minimum.

The end of the collimator tube nearest the lamp was bisected by a vertical thread; a pair of cross threads were placed in the inner end, and a diaphragm with a small aperture at the eye end, of the Nicol tube; and in making the observations care was taken that the image of the vertical thread should coincide with the point of intersection of the two cross threads as seen through the diaphragm.

The observations were made by moving the Nicol tube alternately towards the right and the left. As the image of the flame always remained in the field of view, and the room was completely dark, the angle at which the light was reduced to a minimum could be observed with a fair amount of accuracy.

The amount of light reflected by glass at a perpendicular incidence being small, and a satisfactory diagonal eyepiece not being available, the position of the stage in which the light was incident perpendicularly on the surface of the glass was determined by an indirect method.

The axis of the Nicol tube was first, by means of the diaphragm and cross threads, brought into the same line as that of the collimator tube, and its position read on the divided circle of the goniometer; the tube was then turned through 90°, the glass surface attached to the vertical stage and adjusted, and the stage rotated until the image of the single thread again coincided with the cross threads of the Nicol tube.

The stage reading gave the position in which the light was incident upon the surface at an angle of 45°. To verify the adjustment, the Nicol tube was clamped, first, at an angle differing by + 90°, and then at one differing by − 90°, from its original position.

The reading of the Nicol, when light polarised in the plane of incidence was cut off, had been carefully determined some years previously, and was again verified. To reduce, as far as possible, the errors due to the Nicol (which was of the ordinary construction, with its terminal faces not perpendicular to its geometrical axis), eight readings were made with the prism in one position, and eight more after it had been turned round through 180°.

The actual readings made with the 6·5 mm. plate of crown glass, which had been repolished by Mr. HILGER, were—

°	′	°	′	°	′	°	′
107	06	107	25	106	59	106	55
107	03	107	08	107	02	107	04
107	20	107	09	107	08	106	56
107	27	107	09	107	04	106	55.

The readings made with the other glass surfaces were about as concordant.

Table XV. gives the angles of polarisation as deduced from the means of these readings.

TABLE XV.

Angles of Polarisation.

	°	′	″
Crown glass—			
6·5 mm. plate, repolished by Mr. HILGER	56	22	30
11·5 ,, not repolished	56	14	30
11·5 ,, No. 2, repolished by Mr. HILGER and again with rouge	56	28	30
11·5 ,, ,, ,, with putty powder	56	49	0
15·0 ,, ,, by Mr. HILGER	56	42	30
15·0 ,, ,, with putty powder	56	49	30
18·5 ,, not repolished	56	21	30
18·5 ,, not repolished	56	25	30
24·3 ,, repolished by Mr. HILGER	56	37	30
Wedge used for reflection experiments repolished with putty powder, and subsequently with rouge	56	52	0
Flint glass—			
Face of block—not repolished	58	14	0
,, ,, repolished with putty powder and examined immediately	58	48	0
,, ,, repolished with putty powder and examined after two days	58	44	0

Part II.

When light passes through a transparent plate it is diminished by reflection at the two surfaces, and by "obstruction" within the plate, the cause of obstruction being that a part of the light which has entered the plate is absorbed and, unless the plate be absolutely homogeneous, a part scattered.

If r be the ratio of the light reflected by the first surface, and r' by the second surface, to the light incident upon them, α the coefficient of transmission, and t the thickness of the plate, then the intensity of the transmitted beam is given by the expression $i = \mathrm{I}\rho\rho'\alpha^t$, where $\rho = (1 - r)$ and $\rho' = (1 - r')$.

I, i, and t being known, by eliminating $\rho\rho'$, α can be readily calculated. The value of i depends in the case of coloured media on the refrangibility of the light, but in the case of the two kinds of glass used in these experiments it may be taken to be the same for light of all wave-lengths.

Table XVI. contains the values of α for a thickness of one millimetre of crown and flint glass, obtained by combining in pairs the five values of i for crown glass, and the four values for flint glass, contained in Table VI.

Table XVI.

Values of α.

Crown glass.	Flint glass.
·99685	·99906
·99690	·99884
·99744	·99887
·99729	·99893
·99692	·99837
·99752	·99897
·99750	Mean ·99884
·99809	
·99763	
·99733	
Mean ·99735	

Dr. Robinson, in the paper already mentioned ('Phil. Trans.,' 1869, p. 160), gives the values of n in the expression $i = \mathrm{I}\rho^2 e^{-nt}$, ρ^2 being calculated from Fresnel's formula, and t being the thickness in inches. From the values given by Dr. Robinson for a cylinder of crown, and a prism of flint, glass, both of Messrs. Chance's manufacture, the values of the coefficient α were calculated for a thickness of one millimetre.

	Index for line E.	n.	α.
Crown glass	1·5200	0·0272	0·99893
Flint glass	1·6216	0·0218	0·99914

From the values of α it would appear that these two specimens of glass absorbed somewhat less light than those used in the experiments of which an account is contained in this paper, but Dr. ROBINSON'S results depend on the value of ρ being "calculated *accurately* from FRESNEL'S formula," and if, as seems probable, glass usually reflects less than the theoretical amount of light, the amount absorbed would necessarily appear less than it really was.

From the mean values of α given in Table XVI. the values of ρ (on the assumption that $\rho = \rho'$) were obtained by calculating the values of α^t for the different thicknesses of the two kinds of glass used in these experiments and then introducing these values into the equation $i = I\rho\rho'\alpha^t$.

TABLE XVII.

Values of ρ.

Crown glass.		Flint glass.	
6·5 mm. plate.	·9648	7·0 mm. plate.	·9463
11·5 "	·9636	49·0 "	·9508
15·0 "	·9629	69·5 "	·9461
18·5 "	·9648	91·3 "	·9475
24·3 "	·9642	Mean . .	·9477
Mean . .	·9641		

The value of r for the crown glass is therefore ·0359, and for the flint glass ·0523.

The amount of light which according to theory should have been reflected by the glass was calculated by the expression $\left(\frac{n-1}{n+1}\right)^2$, where n is the index of refraction. These values, and also the amount of the reflected light, as determined directly (see Tables VIII. and IX.), are given in Table XVIII.

TABLE XVIII.

Percentage amount of Light Reflected.

	Observed.		Calculated.
	By transmission.	By reflexion.	
Crown glass	3·59	3·78	4·187
Flint glass	5·23	5·20	5·780

The observed values of the light reflected by the crown glass do not agree quite so well as those for the flint glass; this may be due to the fact that all the measurements with the flint glass were made with one thick block, whilst several pieces of crown glass were used for the transmission experiments, and these plates may have differed slightly both in their composition and in the polish of their surfaces. Making due allowance for this, and for the approximate character of all photometric measurements, the agreement between the results obtained by two entirely distinct methods is, probably, quite as close as could have been anticipated.

The calculated value for both kinds of glass considerably exceeds the observed. As has already been mentioned, the refractive indices, as determined with the large prisms used for the reflection experiments, and with the small prisms, differed slightly; the differences, however, $-$ ·0008 and $+$ ·0055, are not sufficiently great to affect the result to any considerable extent. Thus, at a perpendicular incidence the two theoretical values for the reflected light are 4·187 and 4·176 for the crown glass, and 5·780 and 5·856 for the flint glass, or a difference of 0·011 and 0·076 per cent. of the incident light, a quantity which is, of course, quite inappreciable photometrically.

Both the crown glass and the flint glass had been recently polished, the former by Messrs. CHANCE and the latter by Mr. HILGER, both kinds of glass having been ground with emery and polished with rouge; the crown glass was partially machine-polished but finished by hand, the flint glass entirely hand-polished. The glass surfaces were always well cleaned with wash-leather immediately before being used, as after being left in the laboratory for some days they were usually somewhat tarnished; the films, however, were easily removed, and in no case could any deterioration of the surface be detected.

The effect of repolishing the glass was to increase its reflective power, but Tables X. and XI. show that the two kinds of glass behaved somewhat differently. Immediately after repolishing both reflected more than the theoretical amount of light; but, whilst the crown continued to do so, the reflective power of the flint decreased, and after an interval it reflected the theoretical amount.

TABLE XIX.

	Crown glass.	Flint glass.
Percentage of light reflected before repolishing	3·78	5·20
" " " after repolishing { Immediately	4·27	6·14
" " " after repolishing { After an interval	4·31	5·72
" " " calculated from the observed index of refraction	4·19	5·78

A number of measurements were made of the light reflected by the crown glass at various angles before and after repolishing; the means of the results contained in

Table XII., and also those for crown glass from Table XIX., are given in the second and third columns of Table XX. The amount of light which, according to FRESNEL'S theory, should have been reflected by the glass was calculated for the various incidences by means of the formula

$$J_r^2 = \tfrac{1}{2}\left\{\frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{\tan^2(i-r)}{\tan^2(i+r)}\right\},$$

the values of r being determined from the observed value of the refracting index of the glass; the fourth column contains the results.

By assuming the truth of the theory, the value of the refractive index could, of course, be readily deduced from the amount of light reflected at a nearly normal incidence, this being equal to $\left(\frac{n-1}{n+1}\right)^2$. Before repolishing, the crown glass reflected 3·78 per cent. of the incident light; hence, the value of n would be 1·4842. Assuming that such was the case, the amount of light reflected by the glass at various angles was calculated, and these numbers are given in the fifth column.

TABLE XX.

Percentage amount of Light Reflected by Crown Glass.

Angle of incidence.	Observed. Before repolishing.	Observed. After repolishing.	Calculated. From observed value of n.	Calculated. From calculated value of n.
6° 17′, 6 47, 7 30	3·78	4·29	4·19	
10 0	3·80	4·41	4·19	3·81
20 0	3·77	4·81	4·21	3·81
30 0	3·92	4·55	4·34	3·93
40 0	4·52	5·30	4·77	4·34
50 0	5·53	6·27	5·98	5·52
56 34	7·22	8·24	7·62	7·23
60 0	8·54	9·88	9·16	8·63
65 0	11·16	12·28	12·31	11·75
70 0	15·49	18·28	17·37	16·78
75 0	..	26·33	25·58	

The percentage amount of light reflected before and after repolishing and the amount calculated from the observed index of the glass are represented by the curves on Plate 8, fig. 4, where the abscissæ are the angles of incidence, and the ordinates the percentages. The curve for the values deduced from the index calculated from the amount of light reflected normally is not given, as it coincides so closely with the curve for the glass in its original state that, in order to render the differences visible, it would have been necessary to draw the diagram on a much larger scale.

More observations were made with the glass before repolishing than after, which accounts for the one curve being so much smoother than the other.

These results show (1) that repolishing increased the amount of the reflected light; (2) that before repolishing the amount of light reflected was less than the theoretical amount calculated from the observed index of refraction of the glass, but that in the case of the crown glass, by assuming a value for the index in accordance with the amount of light reflected at a perpendicular incidence, the amount reflected at other angles by the glass before repolishing was given correctly by the formula; (3) that after repolishing the observed amount of light reflected exceeded the calculated amount; (4) that in the case of the flint glass what may be described as the "polishing-effect" passed off in the course of a day or two, and then the theoretical and actual intensities of the reflected light agreed, but that with the crown glass this did not appear to be the case.

The effect of repolishing being to increase the amount of light reflected by the glass, it seemed desirable to ascertain whether repolishing would produce any change, and, if so, whether increase or diminution, in the amount transmitted. If the increase in the reflected light were due to a more perfect surface being obtained, and, therefore, to less light being irregularly reflected or diffused, the intensity of the transmitted beam would certainly not be weakened by repolishing; if, however, it were due to an increase in the refractive index of the surface-layer of the glass, then the intensity of the transmitted beam would be decreased.

Table XXI. contains the means of the values set forth in Tables XIII. and XIV., and in the second column the values for the transmitted light obtained with the same samples of glass two years previously (see Table VI.).

TABLE XXI.

Percentage amount of Light Transmitted by Crown Glass.

	Original determinations.	Not repolished.	Repolished with rouge.	Repolished with putty.
6·5 mm. plate . .	91·50	91·53	90·60	
11·5 ,, . .	90·07	90·28	89·77	89·75
15·0 ,, . .	89·13	89·31	88·61	88·72
18·5 ,, . .	88·51	88·55		
24·3 ,, . .	87·16	86·91	87·77	

Percentage amount of Light Transmitted by Flint Glass.

	Original determinations.	Not repolished.	Repolished with putty and examined	
			Immediately.	After an interval.
7 mm. thick . .	88·83	92·66		
49 ,, . .	85·40	. .	84·55	85·03
69·5 ,, . .	82·57	84·35		
91·3 ,, . .	80·74			

These numbers show that, except with the 24·3 mm. plate of crown glass, the effect of repolishing was to decrease the amount of light transmitted by both kinds of glass; they also show that, whilst the amount transmitted by the crown glass was the same as when it was first examined, the amount transmitted by the flint glass had increased considerably, although both kinds of glass had been kept during the interval wrapped in soft paper and in the same room.

The 24·3 mm. plate of crown glass behaved differently from the others. As is stated in Table XIII., it was first cleaned, and the amount of light it transmitted determined; each surface was then polished for 20 minutes with fine rouge on a silk polisher. This produced no change, and the plate was, therefore, sent to an optician, who returned it with the statement that it had been "polished with rouge on pitch, almost dry, to get a high polish." The moment the glass was placed in the photometer the polish was seen to be defective, the surface being apparently grained, and, as the table shows, its transmissive power was greatly decreased; it was returned to the optician to be again repolished, and then it let through more light than when first tested.

The values of ρ, calculated with the value of α previously obtained (p. 280), are given in Table XXII.

TABLE XXII.

Values of ρ for the Crown Glass.

	Original determinations.	Not repolished.	Repolished with rouge.	Repolished with putty.
6·5 mm. plate . .	·9648	·9650	·9601	
11·5 ,, . .	·9636	·9647	·9617	·9619
15·0 ,, . .	·9629	·9640	·9601	·9608
18·5 ,, . .	·9648	·9644		
24·3 ,, . .	·9642	·9642	[·9672]	
Mean	·9641	·9645	·9609	

Values of ρ for the Flint Glass.

	Original determinations.	Not repolished.	Repolished with putty and examined	
			Immediately.	After an interval.
7 mm. thick . .	·9463	·9665		
49 ,, . .	·9508	. .	·9460	·9482
69·5 ,, . .	·9461	·9562		
91·3 ,, . .	·9475			
Mean	·9477			

These results agree with those obtained by the direct measurement of the reflected light, and show that the effect of the repolishing is to increase the amount of light reflected and to decrease the amount transmitted, and this latter effect must be due to some cause other than a more perfect surface having been obtained.

The polish of the glass plates was examined by holding them close to the aperture in one of the screens of the photometer, and allowing the beam of light from the lamp to pass through the glass, the cross-section of the beam being smaller than the surface of the glass, and all other light being carefully excluded. If the surfaces had been perfectly polished, they would have been quite invisible, but such was not the case, the shadow cast by the edge of the aperture being just visible in all cases.

Examined in this way, there did not seem to be much, if any, difference between the various plates of crown glass, of which two were in their original state and four had been repolished, nor between them and the four faces of the flint block, of which two had been repolished.

The surfaces of the crown and flint glass wedges used for the reflection experiments did not appear to be quite so good; the difference, however, was very slight. The 7 mm. plate of flint glass was much inferior to all the other pieces, the boundaries of the beam of light which passed through it being distinctly visible. When examined in a strong light there appeared to be a slight film on the surface; on continued rubbing with a wash-leather this diminished, and then the surface of the plate, when placed in a beam of light in the dark room, was less visible than before. The inferiority of the surfaces of this plate appeared to be due to the films which had formed on them, and not to any roughness due to imperfect polishing.

The truth of BREWSTER's law, that the tangent of the polarising angle of a substance is numerically equal to its index of refraction, being generally admitted, it seemed desirable to ascertain the values of the polarising angles for the different plates before and after repolishing.

Table XXIII. gives the mean results collected from Table XV. The means show that the effect of repolishing was in all cases to increase the polarising angle, a result which is in accordance with those previously obtained.

TABLE XXIII.—Polarising Angles.

Crown Glass.

	Not repolished.	Repolished with rouge.	Repolished with putty powder.
	° ′ ″	° ′ ″	° ′ ″
	56 14 30	56 22 30	56 49 0
	56 21 30	56 28 30	56 49 30
	56 25 30	56 42 30	56 52 0
		56 37 30	
Mean	56 20 30	56 32 45	56 50 10

Flint Glass.

	Not repolished.	Repolished with putty powder.
	° ′ ″	° ′ ″
	58 14 0	58 48 0
		58 44 0
Mean	58 14 0	58 46 0

Table XXIV. gives the values of the refractive indices of the two kinds of glass as determined directly, and as deduced from the amounts of the reflected and transmitted light, and from the values of the polarising angles, both before and after repolishing. The values obtained in these ways do not agree well together, those deduced from the amount of the transmitted light being considerably the lowest; they show, however, that repolishing increased the theoretical value of the index as determined by three independent methods.

TABLE XXIV.

Values of the Refractive Indices.

		Crown glass.	Flint glass.
Observed		1·5145	1·6330
Calculated from amount of light reflected, determined directly.	Before repolishing	1·4830	1·6280
	After repolishing	1·5220	1·6590* / 1·6290†
Calculated from amount of light reflected, deduced from amount transmitted.	Before repolishing	1·4676	1·5930
	After repolishing	1·4928	1·6055
Calculated from polarising angle.	Before repolishing	1·5017	1·6149
	After repolishing with rouge	1·5136	
	" " putty	1·5293	1·6490

* Immediately after repolishing.

† After an interval.

Sir DAVID BREWSTER stated, many years ago ('Phil. Trans.,' 1815, p. 126), that glass "acquires an incrustation or experiences a decomposition by exposure to the air which alters its polarising angle without altering its general refractive power," and added that by the action of heat alone he had produced a variation of 9° in the polarising angle of flint glass.

SEEBECK ('POGGENDORFF, Annalen,' vol. 20, 1830, p. 27) made a number of determinations of the polarising angles of different kinds of glass, and found that there was considerable difference between the observed values and those calculated from the refractive index, except in the case of surfaces which he himself had ground and polished (with emery and colcothar).

With one specimen of flint glass the difference was originally $-38'$; he then polished it himself, and found that the difference was only $+3'$; after being polished by an optician, the difference became $+28'$.

SEEBECK was of opinion that these differences were due to the treatment which the glass had received whilst being polished and cleaned, and that lapse of time made no change. The only experiments he appears to have made on this latter point were with crown glass, the surface of which, as the experiments here recorded show, does not alter, or at least only alters very slowly.

Lord RAYLEIGH found ('Roy. Soc. Proc.,' vol. 41, p. 275) that repolishing prisms of crown glass caused a considerable increase in the amount of light they reflected; but his experiments do not show that when the prisms were first examined by him they reflected less light than when they were originally polished.

CONCLUSION.

It seems probable that the amount of light reflected by freshly polished glass varies with the way in which it has been polished, and that, if a perfect surface could be obtained without altering the refractive index of the surface-layer, then the amount would be accurately given by FRESNEL's formula, but that usually the amount differs from that given by the formula, being sometimes greater and sometimes less.

The formation of a film of lower refractive index on the glass would account for the defect in the reflected light; but, to account for the excess, it seems necessary to assume that the polishing has increased the optical density of the surface-layer, and the changes produced in the amount of light transmitted and in the angle of polarisation support this view.

After being polished, the surface of flint glass seems to alter somewhat readily, the amount of the reflected light decreasing, and the amount of the transmitted increasing, whilst with crown glass the change, if any, proceeds very slowly.

There is no evidence to show to what particular cause these changes are due.

The values of the transmission coefficients for light of mean refrangibility for the two particular kinds of glass are given, and show that for 1 centimetre the loss by obstruction amounts to 2·62 per cent. with the crown glass, and 1·15 per cent. with the flint glass.

Explanation of Plate 8.

Fig. 1. Double mirror photometer.
A. Photometer board.
B. Scale.
C. Lamp.
DD. Mirrors.
E. Block carrying photometer.
F. Photometer.
G. Wood stops.
H. Screens.
I. Windows in screens.

Fig. 1A. Photometer.
A. Base.
B. Wooden prisms.
CC. Pieces of white paper.
DDD. Windows in casing.

Fig. 2. Polarising photometer.
AA. Nicols.
B. Right-angled prism.
C. Lamp.
DD. Pieces of white paper.
E. Screen.

Fig. 3. Prism photometer.
A. Photometer board.
B. Scale.
C. Lamp.
DD. Pieces of white paper.
E. Block carrying photometric surface.
F. Screen.
G. Prisms.

Fig. 3A. Photometer.
A. Base.
BB. Right-angled prisms.
C. Screw.

Fig. 4. Curves representing the percentage amount of light reflected by crown glass at various angles of incidence.
1. Observed values with the glass in its original condition.
2. Calculated values from observed index of refraction.
3. Observed values with repolished glass.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Fig. 1.

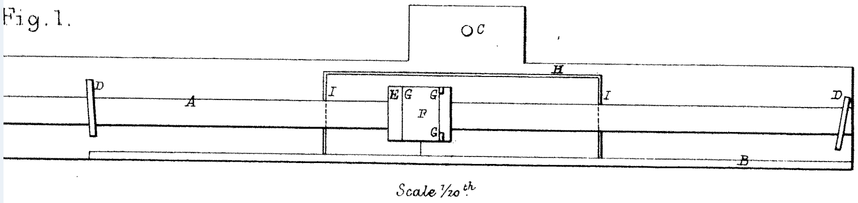


Fig 3.

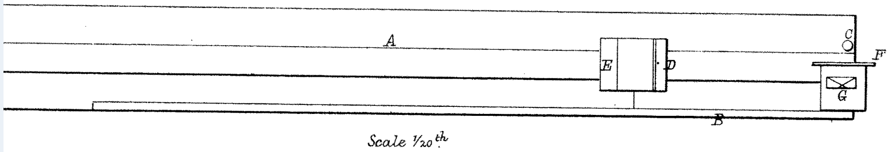
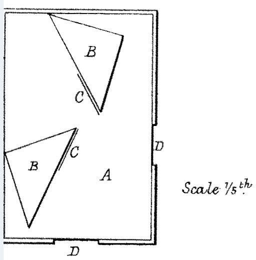


Fig. 1. a



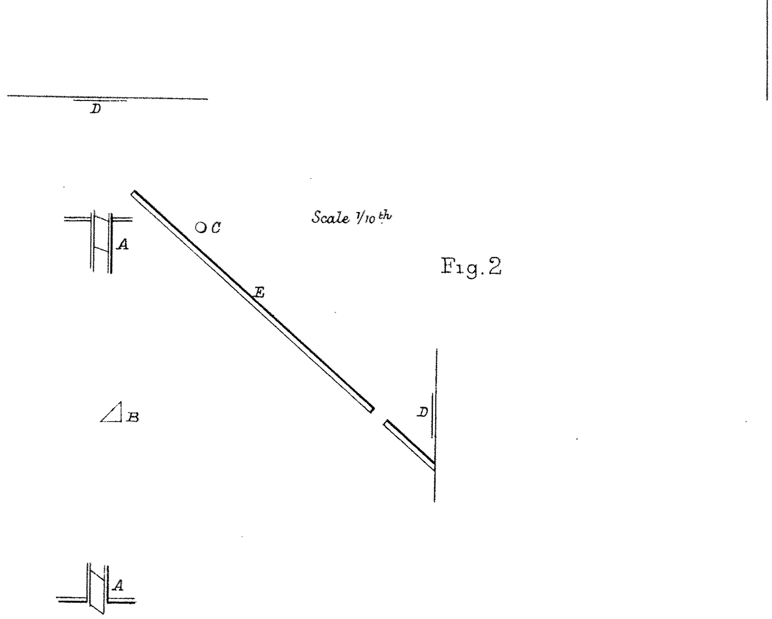


Fig. 2

Fig. 3. a

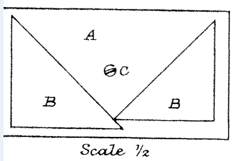
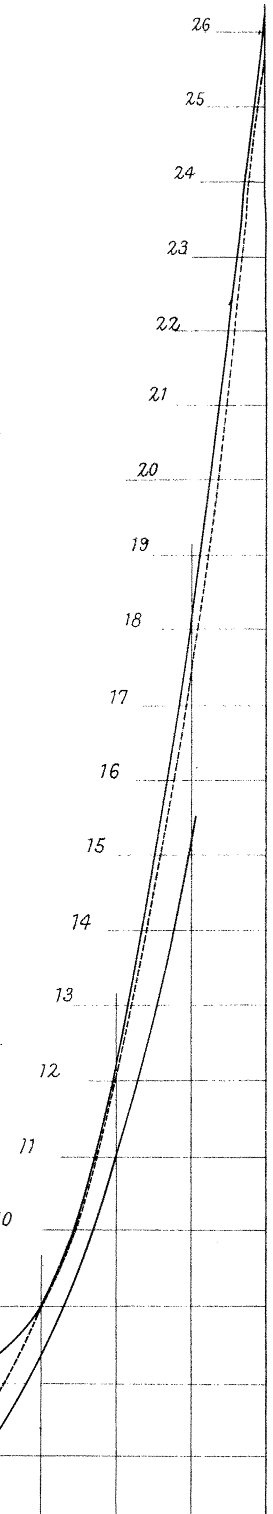
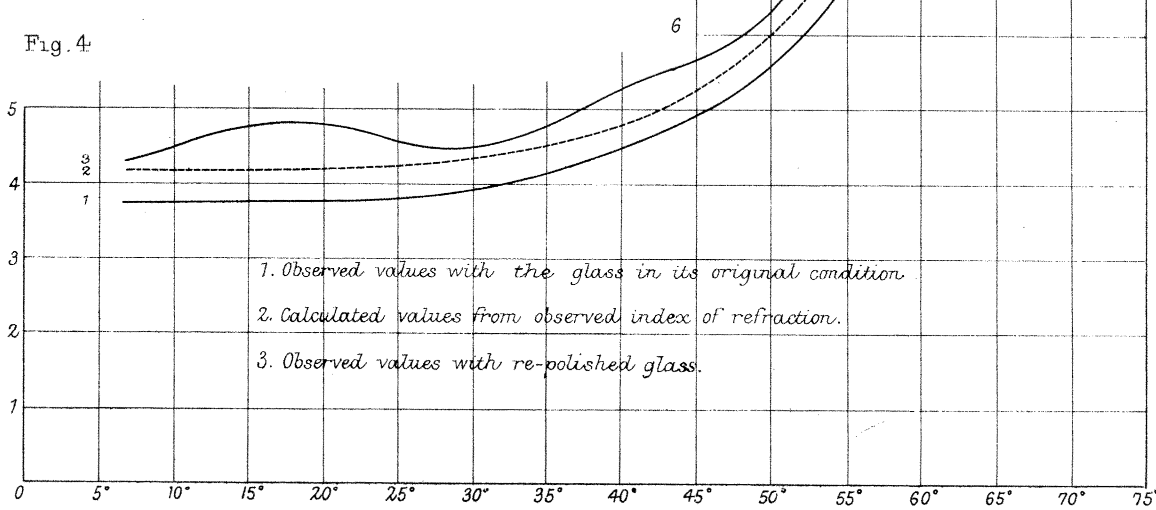


Fig. 4

Per centage amount of light reflected by crown glass at various angles of incidence.

West Newman & C° lith.